Inference for Numerical Data IV

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Review from Last Time

- Hypothesis testing with t-distribution
- When to use:
  - When the sample distribution is nearly normal
  - When the sample sizes are small (a rule of thumb is <30)
- Can be more accurate than a normal distribution
  - When the above 2 conditions are met;

- How to obtain confidence intervals using t-distribution

- How to perform hypothesis testing with t-distribution

Hypothesis Testing with t-Distribution

- T statistic
- For a sample of size n,
  - Estimate sample mean $\bar{X}$ and standard deviation $se(\bar{X})$
  - To test the hypothesis $H_0: \mu = \mu_0$ v.s. $H_A: \mu > \mu_0$
  - A t-statistic can be calculated

$$T = \frac{(\bar{X} - \mu_0)}{se(\bar{X})}$$

The p-value can be assessed by $\Pr(T^* > T)$, where T^* is a random variable with distribution $t_{df=n-1}$

Exercise Problem I

5.16 Working backwards, Part I. A 90% confidence interval for a population mean is (65,77). The population distribution is approximately normal and the population standard deviation is unknown. This confidence interval is based on a simple random sample of 25 observations. Calculate the sample mean, the margin of error, and the sample standard deviation.

Exercise Problem II

5.21 Find the mean. You are given the following hypotheses:

$$H_0 : \mu = 60$$
$$H_A : \mu < 60$$

We know that the sample standard deviation is 8 and the sample size is 20. For what sample mean would the p-value be equal to 0.05? Assume that all conditions necessary for inference are satisfied.

Today's Topic – Two Sample t-test

- Similar to one sample case, it is also useful to derive and use t-distribution to test the difference in the sample mean values
  - When the sample distributions are near normal
  - When the sample sizes are small

- Example:
  - Investigate if ES cells help heart pumping ability among individuals that have a heart attack
    - How to design the experiment
    - What would be the hypothesis that you would like to test?

Procedures

- To test for $H_0: \mu = 0$ v.s. $H_A: \mu \neq 0$
- Closely resembles two-sample test using normal distribution
- Obtain the point estimator for the difference in sample mean values

$\bar{X}_1 - \bar{X}_2$ (as an estimator for $\mu_1 - \mu_2$)
- Obtain the standard error for the sample mean difference point estimate

$$se(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}$$

- Use $T = \frac{\bar{X}_1 - \bar{X}_2}{se(\bar{X}_1 - \bar{X}_2)} \sim t$

$$df = \frac{\left(\frac{\sigma_1^2}{N1} + \frac{\sigma_2^2}{N_2}\right)^2}{\frac{\left(\frac{\sigma_1^2}{N1}\right)^2}{N_1} + \frac{\left(\frac{\sigma_2^2}{N2}\right)^2}{N_2}}$$

How to Obtain Confidence Interval and P-values

- Confidence intervals
  - $\bar{X}_1 - \bar{X}_2 - t_{df} \times se(\bar{X}_1 - \bar{X}_2) \leq \mu_1 - \mu_2 \leq \bar{X}_1 - \bar{X}_2 + t_{df} \times se(\bar{X}_1 - \bar{X}_2)$

Example – Perform Two Sample t-test

Version	n	$\bar{x}$	s	min	max
A	30	79.4	14	45	100
B	27	74.1	20	32	100

Table 5.19: Summary statistics of scores for each exam version.

⊙ **Exercise 5.28** Construct a two-sided hypothesis test to evaluate whether the observed difference in sample means, $\bar{x}_A - \bar{x}_B = 5.3$, might be due to chance.[22]

R Function for Two Sample T-Test

- We can write an R function to calculate the two sample t-statistic
  - So that the same code can be used repeatedly for similar analyses

```
#xA: sample mean for sample A;
#xB: sample mean for sample B;
#sA: standard deviation for sample A;
#sB: standard deviation for sample B;
#nA: sample size for sample A;
#nB: sample size for sample B;

t2 <- function(xA,xB,sA,sB,nA,nB) {
        se=sqrt(sA^2/nA+sB^2/nB)
        z=(xA-xB)/se;
        df=(sA^2/nA+sB^2/nB)^2/((sA^2/nA)^2/nA+(sB^2/nB)^2/nB)
        return(list(z=z,
                se=se,
                df=df));
}
```

ES Cell Transplant Effect on Heart Pumping Ability

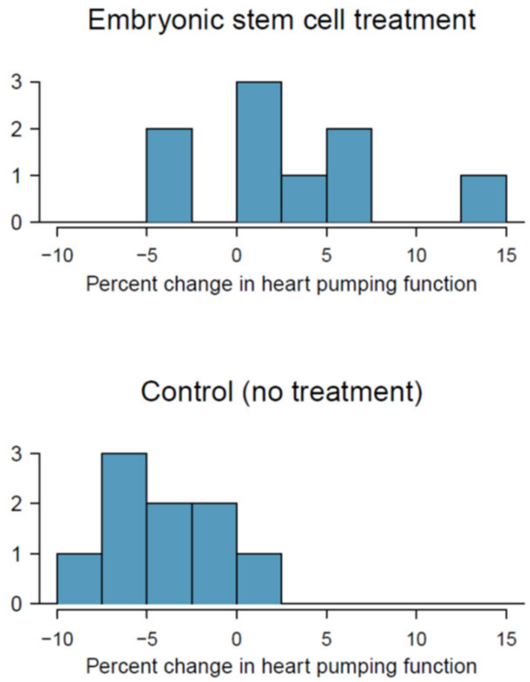


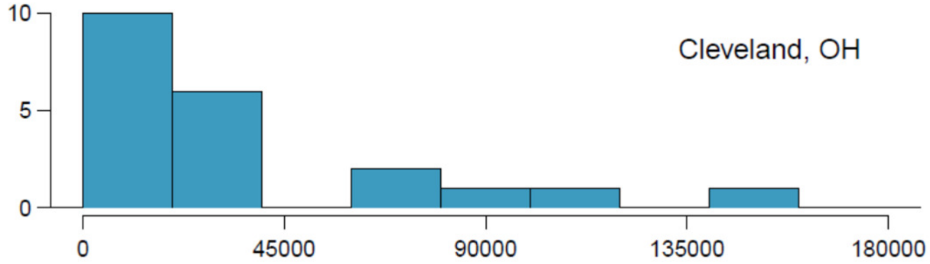
Figure 5.22: Histograms for both the embryonic stem cell group and the control group. Higher values are associated with greater improvement. We don't see any evidence of skew in these data; however, it is worth noting that skew would be difficult to detect with such a small sample.

	n	$\bar{x}$	s
ESCs	9	3.50	5.17
control	9	-4.33	2.76

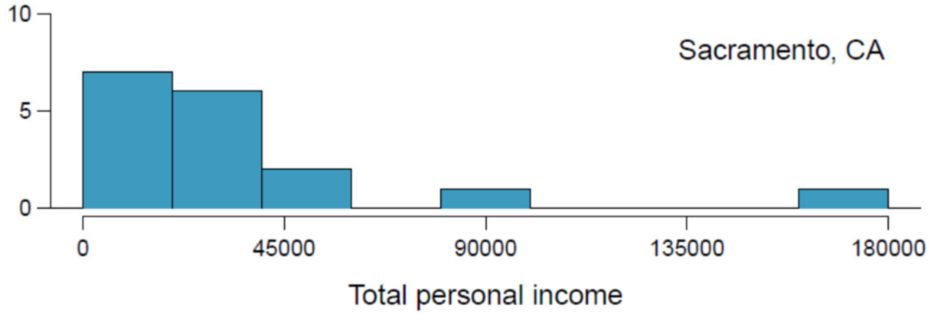
Table 5.21: Summary statistics for the embryonic stem cell data set.

Example I
(make sure to check conditions for t-tests!!)

5.23 Cleveland vs. Sacramento. Average income varies from one region of the country to another, and it often reflects both lifestyles and regional living expenses. Suppose a new graduate is considering a job in two locations, Cleveland, OH and Sacramento, CA, and he wants to see whether the average income in one of these cities is higher than the other. He would like to conduct a t test based on two small samples from the 2000 Census, but he first must consider whether the conditions are met to implement the test. Below are histograms for each city. Should he move forward with the t test? Explain your reasoning.



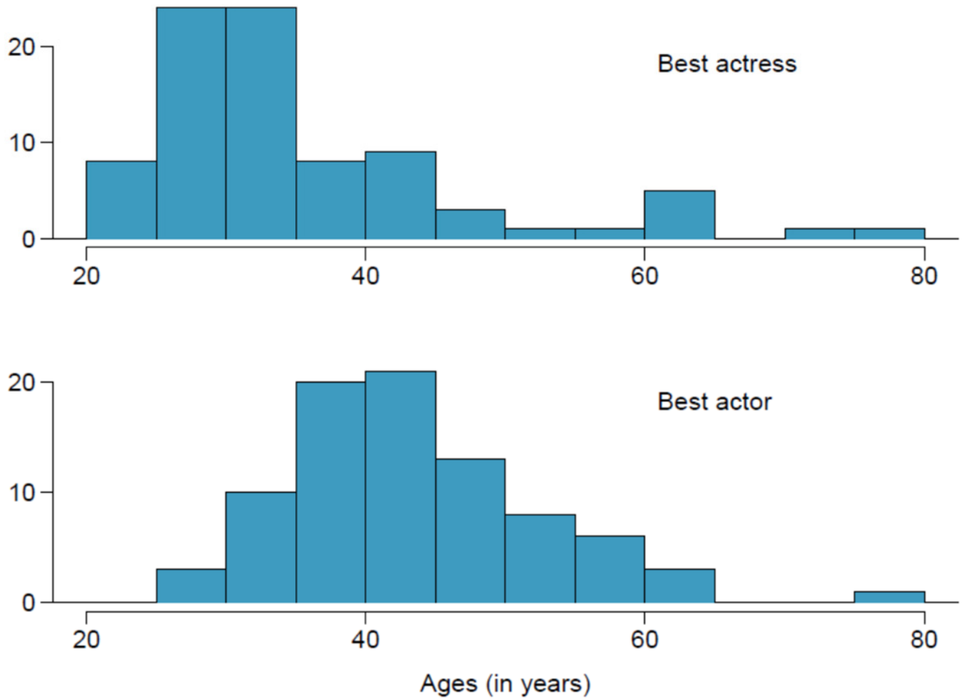
	Cleveland, OH
Mean	\$ 35,749
SD	\$ 39,421
n	21



	Sacramento, CA
Mean	\$ 35,500
SD	\$ 41,512
n	17

Example II

5.24 Oscar winners. The first Oscar awards for best actor and best actress were given out in 1929. The histograms below show the age distribution for all of the best actor and best actress winners from 1929 to 2012. Summary statistics for these distributions are also provided. Is a t test appropriate for evaluating whether the difference in the average ages of best actors and actresses might be due to chance? Explain your reasoning.[41]



	Best Actress
Mean	35.6
SD	11.3
n	84

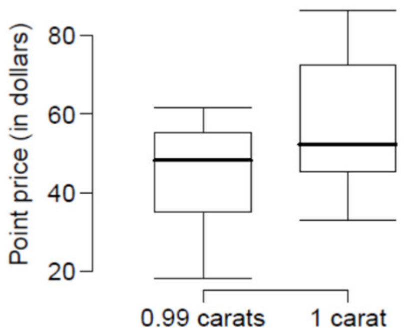
	Best Actor
Mean	44.7
SD	8.9
n	84

Example III

5.26 Diamonds, Part I. Prices of diamonds are determined by what is known as the 4 Cs: cut, clarity, color, and carat weight. The prices of diamonds go up as the carat weight increases, but the increase is not smooth. For example, the difference between the size of a 0.99 carat diamond and a 1 carat diamond is undetectable to the naked human eye, but the price of a 1 carat diamond tends to be much higher than the price of a 0.99 diamond. In this question we use two random samples of diamonds, 0.99 carats and 1 carat, each sample of size 23, and compare the average prices of the diamonds. In order to be able to compare equivalent units, we first divide the price for each diamond by 100 times its weight in carats. That is, for a 0.99 carat diamond, we divide the price by 99. For a 1 carat diamond, we divide the price by 100. The distributions and some sample statistics are shown below.[43]

Conduct a hypothesis test to evaluate if there is a difference between the average standardized prices of 0.99 and 1 carat diamonds. Make sure to state your hypotheses clearly, check relevant conditions, and interpret your results in context of the data.

	0.99 carats	1 carat
Mean	\$ 44.51	\$ 56.81
SD	\$ 13.32	\$ 16.13
n	23	23



Homework Problems

- Exercise 5.18, 5.28, 5.30, 5.32
- Due March 15th, 2016