# Chapter 3. Distribution of random variables

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## 4. Bernoulli distribution

In binomial distribution, we have

(1) N total independent trials

(2) Each trial can be a success or failure

(3) The probability of a success p is the same for each trial

The simplest the case is when the total number of trials is n=1. And the name for this distribution is Bernoulli distribution.

**<u>Example 1</u>**. If X is a random variable that takes value 1 with probability of success p and 0 with probability 1-p, then X is a Bernoulli random variable. Find the mean and Standard deviation of X.

Answer: This is a discrete distribution. The probability distribution for random variable X is as the following:

| | **First Outcome** | **Second Outcome** |
|---|---|---|
| $x_i$ | 1 | 0 |
| $P(X=x_i)$ | p | 1-p |

E(X)=(1)(p)+(0)(1-p)=p

| | First Outcome | Second Outcome |
|---|---|---|
| $x_i$ | 1 | 0 |
| $P(X=x_i)$ | p | 1-p |
| $(x_i-\mu)^2$ | $(1-p)^2$ | $(0-p)^2$ |
| $(x_i-\mu)^2 p(X=x_i)$ | $(1-p)^2 p$ | $(0-p)^2(1-p)$ |

$Var(X) =(1-p)^2p+(0-p)^2(1-p)=(1-p)^2p+p^2(1-p)=p(1-p)$

$\sigma(x)= \sqrt{p(1-p)}$

## 5. Geometric distribution

**<u>Example 2.</u>** If we roll a fair die, what is the probability that the first time we get the "1" is the sixth time when we roll it?

Answer: The probability of **getting the first "1" on the sixth roll** is the probability of **Not getting "1" during the first five rolls** and **getting "1" on the sixth roll**.

So P(getting the first "1" on the sixth roll)= $\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)$

=0.067

**Geometric distribution:**

If the probability of a success in one trial is P and the probability of a failure is 1-p, then the probability of finding the first success in the nth trial is given by

$$\mathbf{(1-p)^{n-1}p}$$

The mean , variance, and standard deviation of this wait time are

$$\mu=\frac{1}{p} \qquad \sigma^2=\frac{1-p}{p^2} \qquad \sigma=\sqrt{\frac{1-p}{p^2}}$$

## 6. Negative Binomial

__Example 3.__ If we roll a fair die, what is the probability that we get the third "1" when we roll the die the sixth time?

Answer: If we get the third "1" on the sixth time, we have to get two "1"s during the Previous five rolls. So the probability of **getting the third "1" on the sixth roll** is the Probability of **" getting two '1's during the first five rolls"** and **" getting '1' on the sixth roll"**.

P(getting the third "1" on the sixth roll)
=P(getting two "1"s during the first five rolls)x P(getting "1" on the sixth roll)

Here P( getting two "1"s during the first five rolls ) is just the binomial distribution.

So P(getting two "1"s during the first five rolls)= $\binom{5}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$

P(getting "1" on the sixth roll)= $\left(\frac{1}{6}\right)$

So the P(getting the third "1"on the sixth roll)= $\binom{5}{2}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^3 = 0.027$

**Negative binomial distribution**

**The negative binomial distribution describes the probability of observing the kth success on the nth trial:**

**P( the kth success on the nth trial)=** $\binom{n-1}{k-1} p^k (1-p)^{n-k}$

Comments on binomial versus negative binomial

(1) Binomial : We have fixed number of independent trial and look for the probability of number of successes

**The P( Exactly k successes in n trials)=** $\binom{n}{k} p^k (1-p)^{n-k}$

(2) Negative binomial: We have fixed number of successes and look for the probability of the number of trials it takes.

**The P(the kth success on the nth trial)=** $\binom{n-1}{k-1} p^k (1-p)^{n-k}$

**Example 4.** If a high school football player Brian makes each 35 yard field goal with probability p=0.8. Prove that the probability Brian kicks fourth successful field goal on the sixth attempt is 0.164.

Answer: This is negative binomial model.

P(kicking fourth successful field goal on the sixth attempt)
=P(kicking 3 successful field goals in the first 5 attempts)
x P( kicking a successful field goal on the sixth attempt)

$$= \binom{5}{3}(0.8^3)(0.2^2)(0.8) = \frac{(5)(4)(3)}{(1)(2)(3)}(0.8^4)(0.2^2) = 0.164$$

**Chapter 3 Homework#5 (due 02/09/16): Page 166 . 3.39. 3.40**