Chapter 2 Probability

1. Definition of Probability
2. Probability of disjoint events
3. Probability of non-disjoint events
4. Probability of complement of an event
5. Probability of independent events
6. Probability of a conditional event
7. Probability of dependent events
8. Tree diagrams
9. Bayes Theorem
10. Mean of random variable
11. Variance of random Variable

1.Definition of probability

- **Probability: Probability of an event is the proportion of times the outcome would occur if we observed the random process an infinite number of times.**

Example 1.

What is the probability of getting an odd number if we roll a dice?

Answer: Probability(1 or 3 or 5)=3/6

2.Probability of disjoint events

- If A_1 and A_2 are disjoint events, then

$P(A_1$ or $A_2)=P(A_1)+P(A_2)$

Similarly if $A_1,A_2,...,A_k,$ are all disjoint events, then

$P(A_1$ or A_2 or ....or $A_k)=P(A_1)+P(A_2)+......+P(A_k)$

Example 2 What is probability of getting an odd number if we roll a dice?

Answer: Probability(an odd number)=P(1 or 3 or 5)

$\qquad$ =P(1)+P(3)+P(5)

$\qquad$ =1/6+1/6+1/6

$\qquad$ =3/6

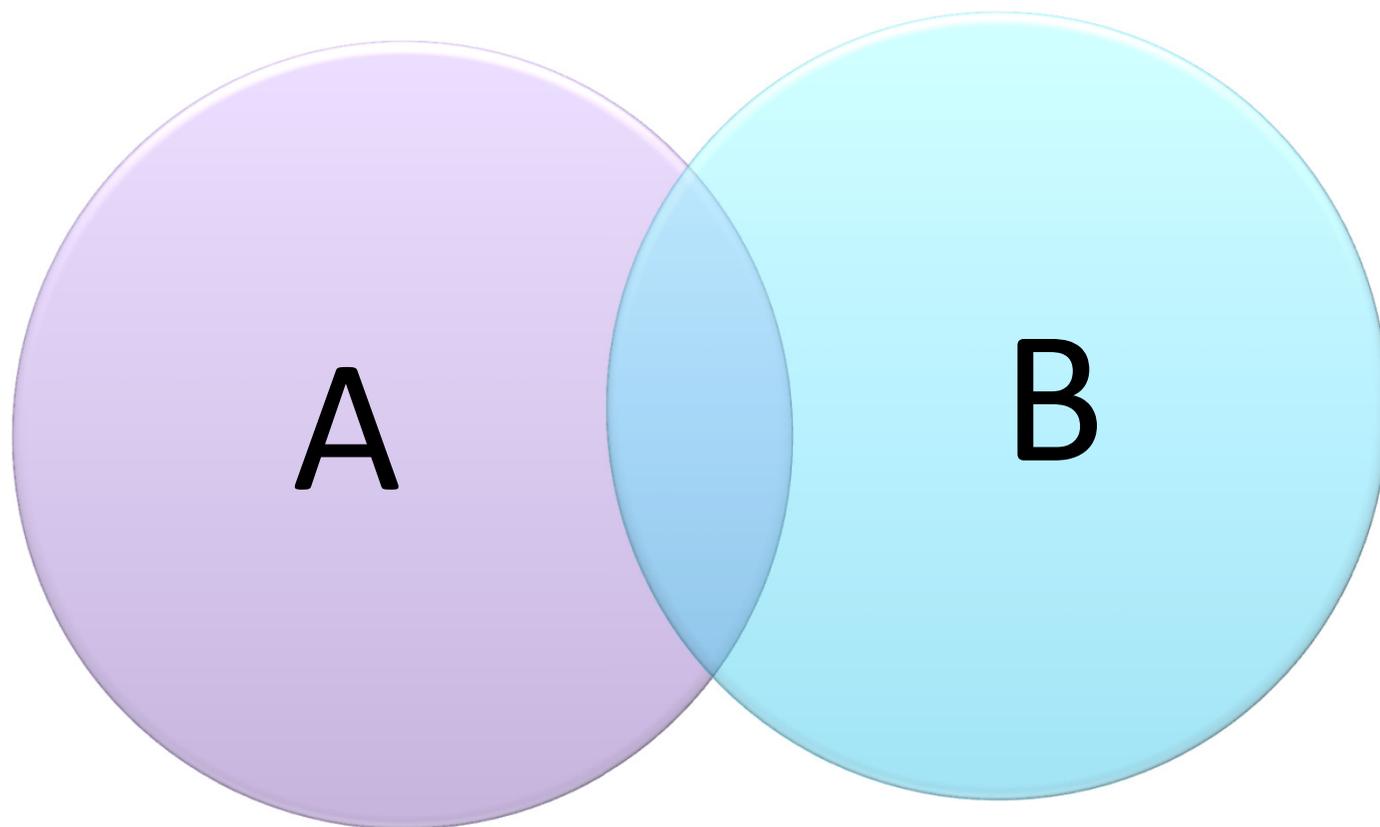# 3. Probability of non-disjoint events

- **If A and B are two non-disjoint events, then**

P(A or B)=P(A)+P(B)-P(A and B)

(This is called General Additional Rule)

Comments: If A and B are disjoint, this rule also apply.

<u>Example 3</u>. Considering a deck of 52 cards, what is the probability of a randomly selected Card is a diamond card or a face card?

Answer: P(diamond or face)=P(diamond)+P(face)-P(diamond and face)

$$=13/52+12/52-3/52$$
$$=22/52=11/26$$

4. Probability of complement of an event

Let A^c be the complement of event A, that is, A^c represents all outcomes that are not in A, then

$P(A^c)=1-P(A)$

Example 4. Consider rolling a die, find the probability of not getting 2.

Answer: P(not 2)=1-P(2)=1-1/6=5/6

5. Probability of independent events

- **Multiplication rule for independent process**

If A and B are independent, then

P(A and B)=P(A)x P(B)

Similarly, if A_1, A_2,....A_k are all independent, then P(A_1 and A_2.....and A_k)=P(A_1)x P(A_2)......x P(A_k)

<u>Example 5.</u> Consider rolling two dice, (a) what is the probability that sum is 6?

        (b)   What is the probability that sum is not 6?

Answer (a) P(Sum is 6)=P(1 and 5)+P(2 and 4)+P(3 and 3)+P(4 and 2)+P(5 and 1)

                                =P(1)x P(5)+ P(2)x P(4)+P(3)x P(3)+P(4)x P(2)+P(5)x P(1)

                                =1/36+1/36+1/36+1/36+1/36

                                =5/36

Answer (b) P(Sum is not 6)=1-P(Sum is 6)=31/36

6. Conditional Probability

The conditional probability of A given condition B is

$$P(A|B) = \frac{P(A \ and \ B)}{P(B)}$$

	inoculated	Not inoculated	total
live	238	5136	5374
die	6	844	850
total	244	5980	6224

Example 6. What is the probability that an inoculated person die from smallpox?

Answer: P(die | inoculation) = 6/244
 If we use the conditional probability formula, we have

$$P(die|inoculated) = \frac{P(die \ and \ inoculated)}{P(inoculated)} = \frac{6/6224}{244/6224} = \frac{6}{244}$$

7. Probability of dependent events

- **General Multiplication Rule : If A and B are two dependent events, then**

$$P(A \text{ and } B) = P(A|B) \times P(B)$$

Comments: (1) when A and B are independent, the above rule still apply. The above equation becomes $P(A \text{ and } B) = P(A) \times P(B)$

(2) The multiplication rule is consistent with conditional probability formula.

Example 7. If P(inoculated)=0.0392, P(live | inoculated)=0.9754

what is the probability that a person was inoculated and lived?

Answer: P(inoculated and lived)=0.0392X0.9754=0.0382

Chapter 2 Homework#1 (due 01/26/16)

1. Please write down your suggestions and expectation for this course.

2. If we draw 5 white balls out of a drum with 69 balls and one red ball out of a drum with 26 red balls. What is the probability of winning the jackpot by matching all five white balls in any order and the red Powerball.