Home Work 3: 4.14, 4.21 and 4.25

4.14 Thanksgiving spending, Part I. The 2009 holiday retail season, which kicked off on November 27, 2009 (the day after Thanksgiving), had been marked by somewhat lower self-reported consumer spending than was seen during the comparable period in 2008. To get an estimate of consumer spending, 436 randomly sampled American adults were surveyed. Daily consumer spending for the six-day period after Thanksgiving, spanning the Black Friday weekend and Cyber Monday, averaged $84.71. A 95% confidence interval based on this sample is ($80.31, $89.11). Determine whether the following statements are true or false, and explain your reasoning.

(a) We are 95% confident that the average spending of these 436 American adults is between $80.31 and $89.11.
Answer: False. The sample mean is always in the confidence interval. The 95% confidence interval covers the population mean (a parameter) with 95% probability.

(b) This confidence interval is not valid since the distribution of spending in the sample is right skewed.
Answer: False. The confidence interval may still be valid when the sample distribution is slightly skewed. Other conditions on the confidence interval are met in this case.

(c) 95% of random samples have a sample mean between $80.31 and $89.11.
Answer: False. Samples of different size may have different confidence intervals. Yet, it is true that the mean value of 95% of the random samples of size 436 lie within the confidence interval.

(d) We are 95% confident that the average spending of all American adults is between $80.31 and $89.11.
Answer: True. According to the definition of the confidence interval, the confidence interval covers the parameter value (which in this case is the average spending of an average American adult) with probability 95%.

(e) A 90% confidence interval would be narrower than the 95% confidence interval since we don't need to be as sure about our estimate.
Answer: True.

(f) In order to decrease the margin of error of a 95% confidence interval to a third of what it is now, we would need to use a sample 3 times larger.
Answer: A sample size of 3 times bigger is not enough, as the standard error equals to $se = \sigma/\sqrt{N}$. To make the confidence interval shrink to one third of what it is now, we would need a sample size of 9 times bigger.

(g) The margin of error is 4.4.
Answer: True. The margin of error is given by $z^{*} \times se$, equivalently, we have (89.11-80.31)/2=4.4.

4.21 Waiting at an ER, Part II. Exercise 4.13 provides a 95% confidence interval for the mean waiting time at an emergency room (ER) of (128 minutes, 147 minutes). Answer the following questions based on this interval.

(a). A local newspaper claims that the average waiting time at this ER exceeds 3 hours. Is this claim supported by the confidence interval? Explain your reasoning.
Answer: No, 3hs= 180 minutes are out of this confidence interval.

(b) The Dean of Medicine at this hospital claims the average wait time is 2.2 hours. Is this claim supported by the confidence interval? Explain your reasoning.
Answer: Yes, 2.2 hs=132 minutes are in CI between 128 minutes and 147 minutes.

(c) Without actually calculating the interval, determine if the claim of the Dean from part (b) would be supported based on a 99% confidence interval?
Answer: Yes because 99% CI covers 95%CI.

4.25 Waiting at an ER, Part III. The hospital administrator mentioned in Exercise 4.13 randomly selected 64 patients and measured the time (in minutes) between when they checked in to the ER and the time they were first seen by a doctor. The average time is 137.5 minutes and the standard deviation is 39 minutes. She is getting grief from her supervisor on the basis that the wait times in the ER has increased greatly from last year's average of 127 minutes. However, she claims that the increase is probably just due to chance.

(a) Are conditions for inference met? Note any assumptions you must make to proceed.
Answer:
Yes. Conditions for inference are
(1) Patients were randomly chosen;
(2) The sample size is more than 30.
In addition, we would also need to verify that the distribution is not highly skewed.

(b) Using a significance level of $\alpha = 0.05$, is the change in wait times statistically significant? Use a two-sided test since it seems the supervisor had to inspect the data before she suggested an increase occurred.
Answer:
(1) The t-test statistic can be calculated as follows:

$$t = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{137.5 - 127}{39}\sqrt{64} = 2.15$$

(2) From t-distribution, t=2.15 has p-value =0.035<0.05, we reject the null hypothesis at $\alpha = 0.05$, and the difference is not likely due to chance.

(c) Would the conclusion of the hypothesis test change if the significance level was changed to $\alpha = 0.01$?
Answer: We cannot reject the null hypothesis at $\alpha = 0.01$